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Magnetohydrodynamics (MHD) Effects on Heat Generation and Joule Heating with Non-Uniform Surface Temperature and Natural Convection Flow over a Vertical Flat Plate

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ABSTRACT

The effects of heat generation on the Magnetohydrodynamics (MHD) natural convection flow along a vertical flat plate with non-uniform surface temperature are analyzed. The governing differential equations are transformed by introducing appropriate non-similarity variables to render them dimensionless, and then numerically resolved. Results for the details of velocity profiles, temperature profiles, local skin frictions, and rate of heat transfer are displayed graphically, and numerical tabulated data for skin friction and rate of local heat transfer are entered in tables with heat generation parameters and Joule Heating parameters.

Keywords: Magnetohydrodynamics (MHD), Natural convection, Joule heating, and Heat generation.

INTRODUCTION:

Natural convection plays a vital part in processes like those involving strong gravitational fields on a global scale or geological processes, and the effects of heat conduction play a significant role in these situations. Both viscous dissipation and pressure work were initially incorporated into the energy equation by Ackroyd, (1962). He demonstrated that the effect of pressure work dominates over that of viscous dissipation. Magnetohydrodynamics (MHD) free convection flow with conduction and Joule heating along a vertical flat plate has been studied by Parveen and Alim, (2011) who have looked at the effects of viscous dissipation and temperature-dependent thermal conductivity. In the context of heat generation Molla *et al.* (2004) investigated the issue of natural convection flow along a vertical wavy surface with a constant surface temperature. Stress analysis of the natural convection flow in a vertical flat plate subjected to joule heating and heat conduction (Alam *et al.,* 2007; Islam *et al*., 2020).

According to Prabhakar and Prabhakararao, (2013) a vertical conical annular porous medium is affected by a temperature variation described by a Power law. Free convection was conducted by Alam *et al.* (2007) using a vertically permeable circular cone, pressure work, and a temperature gradient across the cone's surface. The subject of the effect of a magnetic field on heat and fluid flow over a wavy surface has been explored by Tashtoush and Al-Odat, (2004). Some natural convection flows are affected by pressure stress work and viscous dissipation, as mentioned by Joshi and Gebhart, (1981). MHD-free convection from a vertical plate with power-law change in surface temperature, as studied by Abo-Eldahab and El Aziz, (2005) is subject to the effects of viscous dissipation and Joule heating. Mahajan and Gebhart, (1989) have investigated the role of viscous dissipation in buoyancy-induced flows. Sparrow and Cess, (1961) investigate how magnetic fields influence heat transport via free convection. Shariful and Mohammadein, (2004) investigated the impact of thermal-diffusion and diffusion thermo on heat and mass transport in magnetohydrodynamics. Poots *et al.* (1961) perform magneto hydrodynamics with laminar natural convection flow. The presence of heat generation prompted Molla *et al.* (2006) to explore magnetohydrodynamic natural convection flow on a spherical surface with a homogeneous heat flux. With a thin vertical plate and non-uniform internal heat generation Mendez and Trevino, (2000) study conjugates conduction-natural convection heat transfer. Magneto-hydrodynamic free convection in a strong cross flow field was described by Kuiken, (1970). Free convection from a vertical permeable circular cone with a non-uniform surface heat flux is shown by Hossain and Paul, (2001).

We have investigated the skin friction and local heat transfer coefficient as functions of Prandtl's number Pr, magnetic parameter M, Joule heating parameter J, and heat generation parameter Q, as well as the effects of velocity and temperature fields.

Formulation of the Problem

Take into account laminar free convection flow down a vertical plate placed in a stable environment, where u and v represent the velocity components in the directions, respectively, and where is vertically up and is the coordinate perpendicular to x.

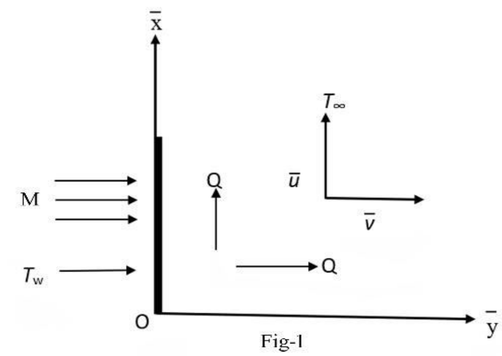

Fig-1

For steady, two-dimensional flow of the boundary layer continuity equation, momentum equation and energy equation including heat generation term and Joule heating term are given below.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial v}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma\beta_0^2 u}{\rho} \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_p}\left(T-T_\infty\right)+\frac{\sigma\beta_0^2 u^2}{\rho} \qquad (3)$$

Where, $\alpha=\frac{k}{\rho C_p}$ is the thermal diffusivity.

Where, for heated upward flows, x is taken vertically up from the active leading edge, and for cooled downward flows, x is taken vertically down. The temperature of quiescent ambient fluid T_∞ at large values of y is taken to be constant.

Where T_w is the temperature at the wall, T is the fluid temperature, ν is the kinematics velocity, β is the fluid thermal expansion coefficient, β_0 is the magnetic field strength, Q_0 is the heat generation coefficient, C_p is the specific heat at the constant pressure, ρ is the fluid density and p is the pressure. The terms for heat generation and Joule heating are the final two terms in the energy equation, respectively.

Equations are too solved subject to the boundary conditions.

$$u=v=o, T=T_w \text{ on } y=0 \qquad (4)$$

$$u \to U, T \to T_\infty \text{ as } y \to \infty$$
$$u = U, T = T_w \text{ at } x = 0$$

Where U is the free stream velocity.
The following generalizations are introduced to obtain the equations in terms of generalized stream and temperature functions $f(x,y)$ and $\phi(x,y)$. Now letting.

$$u(x,y) = \nu\psi_y$$

$$\Rightarrow u = \nu\frac{\partial\psi}{\partial y}$$

$$d(x) = \Delta T = T_w - T_\infty, v(x,y) = -\nu\psi_x \Rightarrow v = -\nu\frac{\partial\psi}{\partial x},\ \varphi(x,y) = \frac{T-T_\infty}{\Delta T}$$

$i.e. T = T_\infty + \varphi\Delta T$

$$\frac{\partial T}{\partial x} = \Delta T\frac{\partial\varphi}{\partial x} \quad = \Delta T\phi_x$$

$|T_\infty, as$ a Constant

Now

$$u = \nu\frac{\partial\psi}{\partial y}$$

$$\therefore \frac{\partial u}{\partial x} = \nu\frac{\partial^2\psi}{\partial x\partial y}, \frac{\partial u}{\partial y} = \nu\frac{\partial^2\psi}{\partial y^2}, \frac{\partial^2 u}{\partial y^2} = \nu\frac{\partial^3\psi}{\partial y^3}$$

Now Equation (2) becomes

$$\Rightarrow \psi_y\psi_{yx} - \psi_x\psi_{yy} = \psi_{yyy} + \frac{g\beta\phi\Delta T}{\nu^2} - \frac{\sigma\beta_0^2}{\rho\nu}\psi_y{}^2 \tag{5}$$

Again, we know

$$\frac{\partial T}{\partial x} = \Delta T\varphi_x + \varphi\,(\Delta T)_x$$
$$\frac{\partial T}{\partial y} = \Delta T\varphi_y$$

T_∞ as a constant and ΔT is a function of x.

$$\frac{\partial^2 T}{\partial y^2} = \nu\frac{\partial^2\psi}{\partial y^2}, P_r = \frac{\mu c_p}{k},$$
$$R_e = \frac{\rho\nu l_c}{\mu} \therefore \frac{1}{\nu} = \alpha\frac{\mu C_p}{k}$$
$$\frac{\mu}{\rho} = \alpha\frac{\mu C_p}{k} \therefore \frac{1}{\rho} = \frac{\alpha}{k}C_p$$

Now equation (3) becomes

$$\Rightarrow \psi_y\phi_x\Delta T + \phi\psi_y(\Delta T)_x - \psi_x\phi_y\Delta T = \frac{k}{\nu\rho c_p}\Delta T\phi_{yy} + \frac{Q_0}{\nu\rho C_p}\Delta T + \frac{\sigma B_0^2}{\rho}\nu\psi_y{}^2 \tag{6}$$

Where $(T_0 - T_\infty)$ A stands for the downstream temperature differential down the x-axis that would have occurred in the absence of the joule heating component. The real-world Grash of the number $Gr_x = Gr_x\phi(0)$ has an association with Gr_x.

Transformation of the Governing Equations

Assuming a similarity variable and a stream function of the following form.

$$x = \xi, \eta = yb(x),\ \psi(x,y) = c(\xi)f(\eta), \phi(x,y) = \phi(\eta)$$

$$\frac{\partial\psi}{\partial y} = c(\xi)\frac{\partial}{\partial y}f(n) = c(\xi)f'(\eta)\frac{\partial\eta}{\partial y} = c(\xi)f'(\eta)\frac{\partial}{\partial y}(yb(\xi))$$

$$\therefore \psi_y = c(\xi) f'(\eta) b(\xi)$$

$$\psi_{yy} = c(\xi) b^2(\xi) f''(\eta),\ \psi_{yyy} = c(\xi) b^3(\xi) f'''(\eta),$$

$$\psi_{xy} = c_\xi(\xi) b'(\xi) f'(\eta) + c(\xi) b_\xi(\xi) f'(n) + c(\xi) b(\xi) f''(\eta) \frac{\partial}{\partial \xi}(\eta)$$

$$\psi_x = c_\xi(\xi) f(\eta) + c(\xi) f'(\eta) \frac{\eta}{b(\xi)} b_\xi(\xi)$$

So the Joule heating term

$$\frac{\sigma \beta_0^2}{\rho} \nu \psi_y{}^2 = \frac{\sigma \beta_0^2}{\rho} \nu c^2(\xi) b^2(\xi) f'^2(\eta)$$

Temperature profiles

$$\varphi_y = \varphi'(\eta) b(\xi)$$

$$\varphi_{yy} = \varphi''(\eta) b^2(\xi)$$

$$\varphi_x = \frac{\eta}{b(\xi)} b_\xi(\xi).\varphi'(\eta)$$

Therefore, the momentum equation

$$\Rightarrow \psi_y \psi_{yx} - \psi_x \psi_{yy} = \psi_{yyy} + \frac{g\beta \phi \Delta T}{\nu^2} - \frac{\sigma \beta_0^2}{\nu^2} \psi_y$$

$$\Rightarrow \frac{f'^2(\eta) - f(\eta) f''(\eta)}{b(\xi)} c_\xi(\xi) + \frac{c(\xi) b_\xi(\xi)}{b^2(\xi)} f'^2(\eta) = f'''(\eta) + \frac{g\beta\varphi\Delta T}{c(\xi) b^3(\xi) \nu^2} - \frac{\sigma \beta_0^2}{\mu b^2} f'$$

Therefore, the momentum equation is

$$\Rightarrow f'''(\eta) + \frac{g\beta\phi\Delta T}{c(\xi) b^3(\xi) \nu^2} + \frac{c_\xi(\xi)}{b(\xi)} f(\eta) f''(\eta) - \left[\frac{c_\xi(\xi)}{b(\xi)} + \frac{c(\xi) b_\xi(\xi)}{b^2(\xi)}\right] f'^2(\eta) + - \frac{\sigma \beta_0^2}{\mu b^2} f' = 0 \qquad (7)$$

Also, the energy equation

$$\psi_y \phi_x \Delta T + \phi \psi_y (\Delta T)_x - \psi_x \phi_y \Delta T = \frac{1}{P_r} \Delta T \phi_{yy} + \frac{Q_o}{\nu \rho C_p} \phi \Delta T + \frac{\sigma \beta_0^2}{\rho} \nu \psi_y{}^2$$

$$\Rightarrow c(\xi) \frac{b_\xi(\xi)}{b^2(\xi)} f'(\eta) \varphi'(\eta) + \varphi\, c(\xi) \frac{f'(\eta)(\Delta T)_\xi}{b(\xi) \Delta T} - \frac{c_\xi(\xi) f(\eta) \varphi'(\eta)}{b(\xi)} - \eta \frac{c(\xi) b_\xi(\xi)}{b^2(\xi)} f(\eta) \varphi(\eta)$$

$$= \frac{1}{p_r} \varphi''(\eta) + \frac{Q_0}{b^2(\xi) \nu \rho C_p} \varphi + \frac{\sigma \beta_0^2}{\rho \Delta T} \nu\, c^2(\xi) f'^2(\eta)$$

Divide $\Delta T b^2(\xi)$

$$\Rightarrow \frac{1}{P_r} \phi''(\eta) + \frac{c_\xi(\xi)}{b(\xi)} f(n) \phi'(\eta) - \frac{c(\xi)(\Delta T)_\xi}{b(\xi) \Delta T} f'(\eta) \phi(n) \qquad (8)$$

$$- \frac{Q_o}{b^2(\xi) \nu \rho C_p} \phi + \frac{\sigma \beta_0^2}{\rho \Delta T} \nu c^2(\xi) f'^2(\eta) = 0$$

The heat generation term and the Joule heating effect are the final two terms in the energy equation (8). It is widely understood that when these concepts are ignored,, similarity solutions exist for a power law $\Delta T = N x^n = N \xi^n$ surface (at η=0) temperature distributions.

$$c(x) = 4\left(\frac{1}{4} Gr_x\right)^{\frac{1}{4}}, b(x) = \frac{1}{x}\left(\frac{1}{4} Gr_x\right)^{1/4}, Gr_x = \frac{g\beta x^3 n x^n}{\nu^2}$$

$$\therefore c_x = \frac{d}{dx}[4(\frac{1}{4} Gr_x)^{1/4}] = \frac{d}{dx}[4(\frac{1}{4}\frac{g\beta x^3 n x^n}{\nu^2})^{1/4}] = 4.\left(\frac{1}{4}\frac{g\beta n}{\nu^2}\right)^{1/4} \frac{d}{dx}(x^{\frac{n+3}{4}})$$

$$=\frac{n+3}{x}\left(\frac{1}{4}\frac{g\beta n}{\nu^2}x^n x^3\right)^{1/4}=\frac{n+3}{x}\left(\frac{1}{4}Gr_x\right)^{1/4}$$

$$b_x=\frac{d}{dx}\left[\frac{1}{x}\left[\frac{1}{4}\frac{g\beta x^3 n x^n}{\nu^2}\right]^{1/4}\right]=\left[\frac{1}{4}\frac{g\beta n}{\nu^2}\right]^{1/4}\frac{n-1}{4}x^{\frac{n+3}{4}-2}=\frac{n-1}{4x^2}\left(\frac{1}{4}Gr_x\right)^{1/4}$$

Now

$$(\Delta T)_x=\frac{d}{dx}(Nx^{\eta})=nNx^{\eta-1}=\frac{n}{x}\Delta T$$

Then for the momentum equation

$$f'''(\eta)+\frac{g\beta\varphi\Delta T}{c(\xi)b^3(\xi)\nu^2}+\frac{c_\xi(\xi)}{b(\xi)}f(\eta)f''(\eta)-\left[\frac{c_\xi(\xi)}{b(\xi)}+\frac{c(\xi)b_\xi(\xi)}{b^2(\xi)}\right]f'^2(\eta)+\frac{\sigma\beta_0^2}{\mu b^2}f'=0 \tag{9}$$

$$\Rightarrow f'''(\eta)+(n+3)f''(\eta)-2(n+1)f'^2(\eta)+\phi\frac{g\beta N\xi^n\xi^3}{\nu^2}\frac{\nu^2}{g\beta N\xi^n\xi^3}+\frac{\sigma\beta_0^2}{\rho}\frac{2x^2}{(g\beta N)^{1/2}x^{\frac{n+3}{2}}}f'=0$$

$$\Rightarrow f'''(\eta)+(n+3)f(\eta)f''(\eta)-2(n+1)f'^2(\eta)+\phi(\eta)+H_a f'=0 \tag{10}$$

Where, $H_a=\dfrac{2\sigma\beta_0^2\xi^2}{\mu\left(Gr_\xi\right)^{\frac{1}{2}}}$, Hartman number related to MHD.

And energy equation

$$\frac{1}{p_r}\phi''(\eta)+\frac{\frac{n+3}{\xi}\left(\frac{1}{4}Gr_\xi\right)^{1/4}}{\frac{1}{\xi}\left(\frac{1}{4}Gr_\xi\right)^{1/4}}f(\eta)\phi'(\eta)-\frac{4\left(\frac{1}{4}Gr_\xi\right)^{1/4}\frac{n}{\xi}\Delta T}{\frac{1}{\xi}\left[\frac{1}{4}Gr_\xi\right]^{1/4}\Delta T}f'(\eta)\phi(\eta)$$

$$-\frac{Q_o}{b^2(\xi)\nu\rho C_p}\phi+\frac{\sigma\beta_0^2}{\rho\Delta T}\nu 4^2\left(\frac{1}{4}Gr_\xi\right)^{\frac{1}{2}}f'^2(\eta)=0$$

$$\frac{1}{p_r}\varphi''(\eta)+(n+3)f(\eta)\varphi'(\eta)-4nf'(\eta)\varphi(\eta)-\frac{Q_o}{b^2(\xi)\nu\rho C_p}\varphi+\frac{16\sigma\beta_0^2}{\rho\Delta T}\nu$$

$$\left(\frac{1}{4}Gr_\xi\right)^{\frac{1}{2}}f'^2(\eta) \tag{11}$$

Therefore Joule heating term are included, $f(\eta)$ and $\phi(\eta)$ are functions of η, p_r and ξ^n for the power law case.

To retain Joule heating terms to the first order $J(\xi)$ are chosen $J(\xi)=\dfrac{16\sigma\beta_0^2}{\rho\Delta T}\nu\left(\dfrac{1}{4}Gr_\xi\right)^{\frac{1}{2}}$

So, the equation (11) becomes

$$\phi''(\eta)+p_r[(n+3)f(\eta)\phi'(\eta)-4nf'(\eta)\phi(\eta)$$

$$-\frac{Q_o}{b^2(\xi)\nu\rho C_p}\phi(\eta)+J(\xi)f'^2(\eta)]=0$$

$$\phi''(\eta)+p_r[(n+3)f(\eta)\phi'(\eta)-4nf'(\eta)\phi(\eta)-Q\phi(\eta)+J(\xi)f'^2(\eta)]=0 \tag{12}$$

Where, $Q=\dfrac{Q_0\,\xi^2}{\nu\,\rho\,C_p\,(\frac{1}{4}Gr_\xi)^{\frac{1}{2}}}$

Q is the dimensionless heat generation parameter.

Equations (1.10) and (1.12) are numerically integrated for the vertical surface case, with the following boundary conditions

$$\eta=0:\quad f(0)=0,\quad f'(0)=0,\quad \phi(0)=1; \tag{13}$$

$$\eta\to\infty:\quad f'(\infty)\to 0,\quad \phi(\infty)\to 0,$$

Skin Friction Coefficient and Heat Transfer Coefficient

For the flat surface the heat flux is given by,

$$q''(x) = -k \frac{\partial T}{\partial y}\Big|_{y=0}$$

We know, $T = T_\infty + \phi \Delta T$

$$\frac{\partial T}{\partial y} = b\Delta T \varphi'(\eta)$$

$$\frac{\partial t}{\partial y}\Big|_{y=0} = b\Delta T \phi'(0)$$

The proper $b(x)$ and $\Delta T = d(x)$ give the following heat flux for the power law case.

$$q''(x) = -k\,\Delta T \phi'(0)]b(\xi)$$

Where, $\Delta T = d(x) = Nx^n, b(x) = \frac{1}{x}[\frac{1}{4}Gr_x]^{1/4}, Gr_x = \frac{g\beta x^3 N x^n}{\nu^2}$

Heat transfer coefficient (Nusselt Number)

$$Nu_x = \frac{q''(x)}{(T_0 - T_\infty)}\frac{x}{k} = \frac{h_x x}{k}$$

Local Nusselt Number, $Nu_\xi = -(\frac{1}{4}Gr_\xi)^{\frac{1}{4}}\varphi'(0)$

The viscous stress $\tau(\xi)$ and the Skin friction coefficient C_{f_ξ} defined on a convection velocity, $U = U_c = \nu\, c(\xi) b(\xi) f'_{\max} \propto \nu\, c(\xi) b(\xi)$ are know from $f(\eta)$

$$\therefore \tau(\xi) = \mu\nu\, c(\xi) f''(\eta)\{b(\xi)\}^2$$

$$\therefore \tau(\xi) = \mu \frac{\partial u}{\partial y}\Big|_{y=0} = \mu\nu\, c(\xi) b^2(\xi) f''(0)$$

When $y = 0, \eta = 0$

Therefore, Skin friction coefficient

$$C_{f\xi} = \frac{2\,\mu\,\nu\, c(\xi) b^2(\xi) f''(0)}{\rho\nu^2\, c^2(\xi) b^2(\xi) f'^2_{\max}(0)} \propto \frac{2\,\mu\,\nu\, c(\xi) b^2(\xi) f''(0)}{\rho\nu^2\, c^2(\xi) b^2(\xi)}$$

$$C_{f\xi} = \frac{f''(0)}{2(\frac{1}{4}Gr_\xi)^{\frac{1}{4}}}$$

RESULTS AND DISCUSSION:

The Effects of heat generation on MHD Natural Convection Flow along a Vertical Flat Plate with Non-Uniform Surface Temperature have been analyzed in this present work. The settings of the parameter controlling heat generation in this simulation are Q = 1.50, 1.20, 0.90, 0.50, magnetic parameter M = 0.50, 0.40, 0.30, 0.20, 0.10, joule heating parameter J = 0.50, 0.40, 0.30, 0.20, 0.10, and Prandlt's number Pr = 7.00, 3.00, 1.00, 0.70. If we know the functions $f(\xi, \eta)$, $\phi(\xi, \eta)$ and their derivatives for the various heat generating parameter values Q, magnetic parameter M, Prandtl's number Pr, and the Joule heating parameter J. **Fig. 2–5** show the velocity and temperature curves obtained from the solution . Also, the local skin friction $f''(\xi, 0)$ and local heat transfer $\phi'(\xi, 0)$ obtained from figure 6 to 9. **Fig. 2a and 2b** display results for the effect of the heat generation parameter Q = 1.50, 1.20, 0.90, 0.50 for various values of the controlling parameter M = 0.50, Pr = 0.70, J = 0.50, n = 1.00 on the velocity profile $f'(\xi, \eta)$ increases with the increase of heat generation parameter Q which indicates that the heat generation parameter increases the fluid motion. In **Fig. 2b** it is shown that the temperature profile $\phi(\xi, \eta)$ increase for increasing values of Q with another controlling parameter. The effects of magnetic parameter or Hartmann number M = 0.50, 0.40, 0.30, 0.20, 0.10 for Q = 1.50, Pr = 0.70, J = 0.50 and n = 1.00 on the velocity profiles and temperature profiles are shown in **Fig. 3a and 3b**. **Fig. 3a and 3b** show the impacts of magnetic parameter M on the velocity and temperature curves, respectively.

The velocity profiles are found to decrease as the value of M increases, whereas the temperature profiles increase as the value of M increases. In **Fig. 4a and 4b** illustrate the effect of Joule heating parameter J = 0.50, 0.40, 0.30, 0.20, 0.10 for M = 0.50, Pr=0.70, Q = 1.50 and n = 1.00 on the velocity profiles and temperature profiles are shown in **Fig. 4a and 4b**. **Fig. 4a and 4b** show the impact of the Joule heating parameter J on the velocity and temperature curves, respectively. The velocity profiles in this image grow as the value of J increases, and the temperature profiles also increase with the value of the Joule heating parameter J increases.

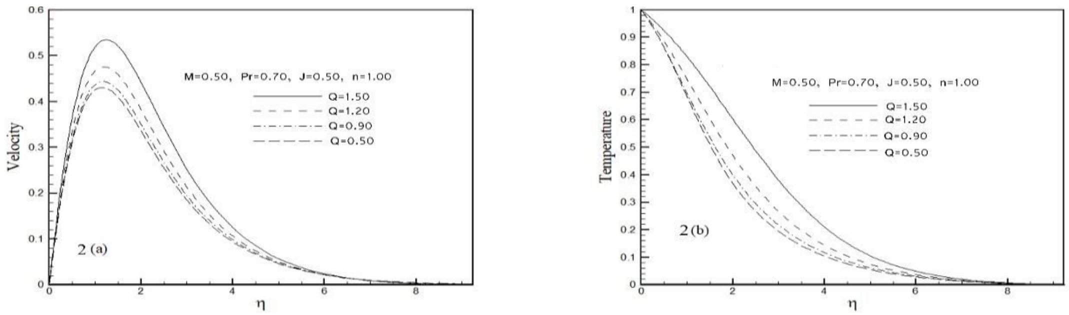

Fig. 2a and 2b: Displayed for M = 0.50, Pr = 0.70, J = 0.50, n = 1.00 and Q = 1.50, 1.20, 0.90, 0.50.

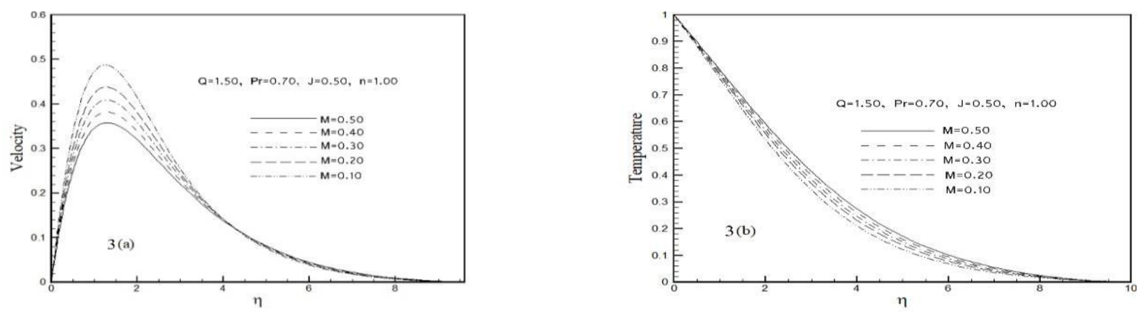

Fig. 3a and 3b: Displayed for Q = 1.50, Pr = 070, J = 0.50, n = 1.00 and M = 0.50, 0.40, 0.30, 0.20, 0.10.

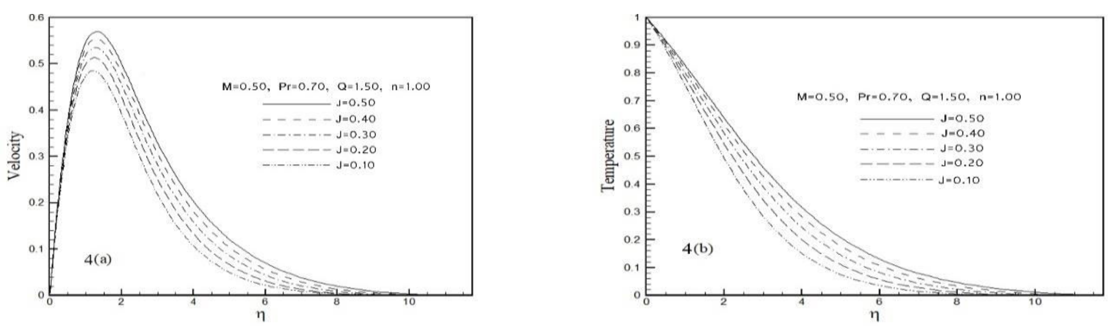

Fig. 4a and 4b: Displayed for M = 0.50, Pr=0.70, Q = 1.50, n = 1.00 and J = 0.50, 0.40, 0.30, 0.20, 0.10.

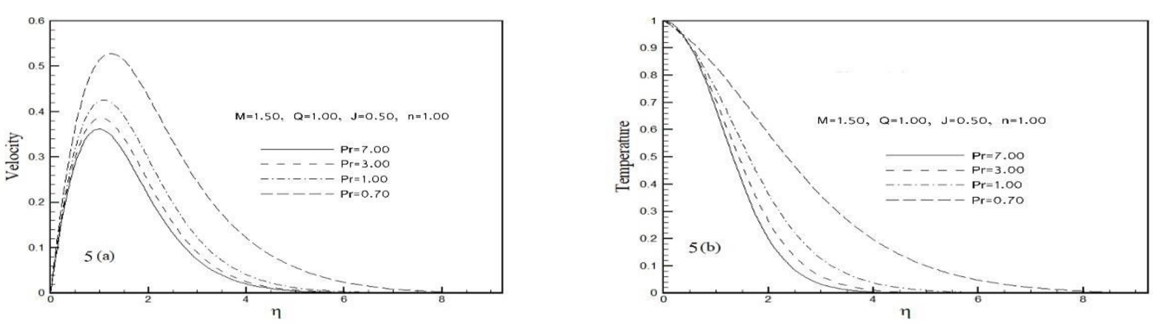

Fig. 5a and 5b: Displayed for M = 1.50, Q = 1.00, J = 0.50, n = 1.00 and Pr = 7.00, 3.00, 1.00, 0.70.

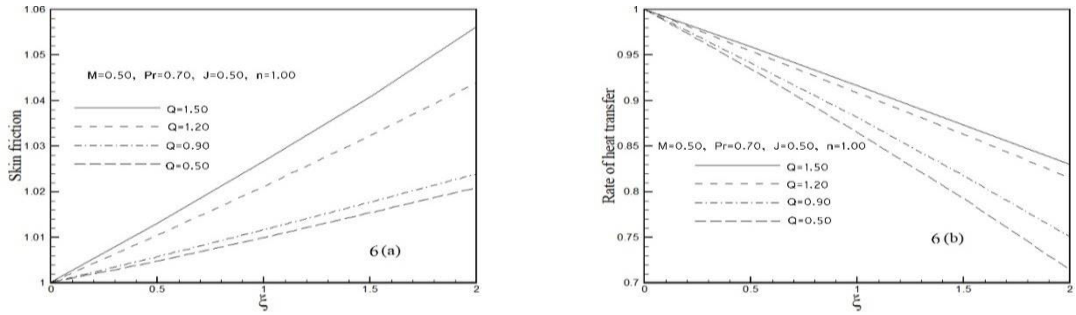

Fig. 6a and 6b: Displayed for M = 0.50, Pr = 0.70, J = 0.50, n = 1.00 *and* Q = 1.50, 1.20, 0.90, 0.50.

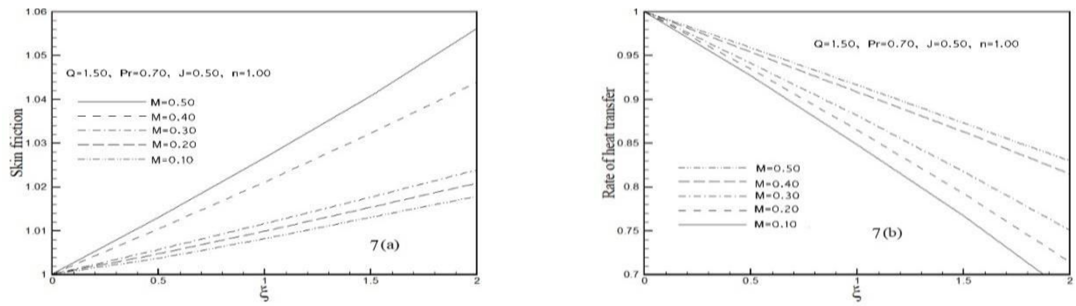

Fig. 7a and 7b: Displayed for Q = 1.50, Pr = 0.70, J = 0.50, n = 1.00 and M = 0.50, 0.40, 0.30, 0.20, 0.10.

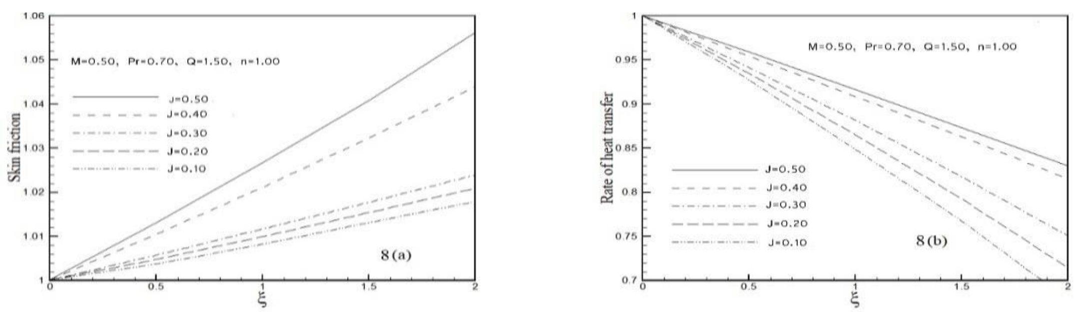

Fig. 8a and 8b: Displayed for M = 0.50, Pr = 0.70, Q = 1.50, n = 1.00 *and* J = 0.50, 0.40, 0.30, 0.20, 0.10.

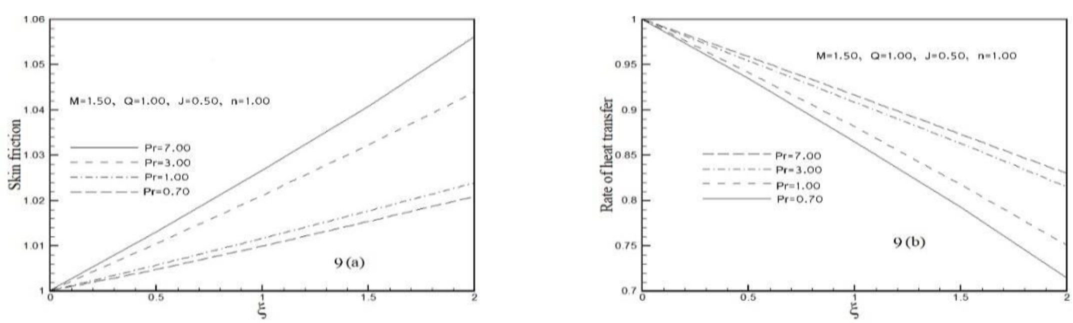

Fig. 9a and 9b: Displayed for M = 1.50, Q = 1.00, J = 0.50, n = 1.00 and Pr = 7.00, 3.00, 1.00, 0.70.

Fig. 5a depicts the velocity profiles for different values of prandtl's number, Pr = 7.00, 3.00, 1.00, 0.70 while the other controlling parameter M = 1.50, Q = 1.00, J = 0.50, n = 1.00. Corresponding distribution of the temperature profiles $\phi(\xi, \eta)$ in the fluids is shown in **Fig. 5b**. **Fig. 5a** shows that increasing the prandtl's number decreases the fluid velocity. **Fig. 5b**, on the other hand, shows that when the prandtl's number Pr increases, the temperature profiles within the boundary layer drop. Numerical values of the skin friction $f''(\xi, 0)$ and the local heat transfer coefficient $\phi'(\xi, 0)$ are depicted graphically in **Fig. 6a** and **6b** respectively against ξ for different values of the heat generation parameter Q = 1.50, 1.20, 0.90, 0.50 for M = 0.50, Pr = 0.70, J = 0.50, n = 1.00. It is seen from fig 6a that the skin friction $f''(\xi, 0)$ increases when the heat generation parameter Q increases. It is also observed in fig 6b, local heat transfer coefficient $\phi'(\xi, 0)$ increases while the heat generation parameter Q increases. The effects of magnetic parameter or Hartmann number M = 0.50, 0.40, 0.30, 0.20, 0.10 for Q = 1.50, Pr = 0.70, J = 0.50 and n = 1.00 on the local skin friction coefficient $f''(\xi, 0)$ and local heat transfer coefficient $\phi'(\xi, 0)$ are shown in **Fig. 7a and 7b**. **Fig. 7a and Fig. 7b** it is evident that the increasing value of M leads to increase the skin friction coefficient $f''(\xi, 0)$ and decrease local heat transfer coefficient $\phi'(\xi, 0)$. The variation of skin friction and heat transfer for different values of Joule heating parameter J = 0.50, 0.40, 0.30, 0.20, 0.10 against ξ for M = 0.50, Pr=0.70, Q = 1.50 and n = 1.00 are shown in figure 2.8a and 2.8b. Figure 2.8a and figure 2.8b it is found that the increasing values of Joule heating parameter J leads to increase both the skin friction coefficient $f''(\xi, 0)$ and local heat transfer coefficient $\phi'(\xi, 0)$. From **Fig. 2a**, it is observed that increase of the value of prandtl's number Pr = 7.00, 3.00, 1.00, 0.70 leads to increase of the value of skin friction Coefficient $f''(\xi, 0)$ and local heat transfer coefficient $\phi'(\xi, 0)$ also increases shown in figure 2.9b for the same values of Prandlt's number Pr against ξ M = 1.50, Q = 1.00, J = 0.50, n = 1.00.

CONCLUSION:

From the present investigation, the following conclusions may be drawn. Increased values of the heat generation parameter Q leads to increase the velocity profiles, the temperature profiles, the local skin friction coefficient $f''(x, 0)$ and the local heat transfer coefficient while M = 0.50, Pr = 0.70, J = 0.50, Vd = 0.60, n = 1.00 . The effect of magnetic parameter or Hartmann Number M is to increase the temperature profiles, the local skin friction coefficient $f''(x, 0)$ but the local heat transfer coefficient, the velocity profiles decrease with the increasing value of M. Increased values of the Joule heating parameter J led to increase the velocity profiles, the skin friction coefficient, the temperature profile and the local heat transfer coefficient. It has been observed that the skin friction and the local heat transfer coefficient increases for increasing value of Pr but the temperature profile, the velocity profile decreases with the increasing value of Pr.

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CONFLICTS OF INTEREST:

The authors declare that there are no conflicts of interest regarding the publication of this research article.

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